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APPROXIMATIONS TO THE INVERSE CUMULATIVE NORMAL FUNCTION FOR US--ETC(U)
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(6) APPROXIMATIONS TO THE INVERSE CUMULATIVE NORMAL FUNCTION FOR USE ON HAND CALCULATORS.

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Intended for use on a pocket calculator, Page (1977) gives an approximation to the cumulative normal function

$$F(x) = \int_{-\infty}^{x} (2\pi)^{-1/2} \exp(-u^2/2)du$$

and its inverse $x = F^{-1}(p)$. Given here are two other approximations to the inverse function which are easier to apply.. (It should be noted that more accurate, but also more complicated, approximations are well known. See, for example, Abramowitz and Stegun (1972), equations 26.2.22 and 26.2.23.)

Given a fractile $p \geq 1/2$, Page's approximation to x, denoted x', is calculated in a three step procedure:

1. Set $y = \ell n\{p/(1 - p)\}/1.59577$
2. Set $u = [\{y + (y^2 + 3.31316)^{1/2}\}/.08943]^{1/3}$
3. Set $x' = u - (.134145u)^{-1}$.

For $p < 1/2$, $1 - p$ is used and the sign of x' is reversed.

A simpler approximation, based on results from Ramberg and Schmeiser (1972), is

$$x^* = [p^{.135} - (1 - p)^{.135}]/.1975$$

which is valid for any fractile p.

Additional accuracy for large values of x may be obtained using

$$\hat{x} = .2 + [p^{.14} - (1 - p)^{.09}]/.1596$$

where $p > 1/2$. For $p < 1/2$, $1 - p$ is used and the sign of $\hat{x}$ is reversed.

Table 1 compares the accuracy of the approximations x', x^*, and $\hat{x}$. x' and x^* exhibit similar behavior, both being least accurate in the tail. x' is more accurate than x^* for most values of x, but for many applications the simpler x^* may be suitable. The most consistent accuracy is obtained with $\hat{x}$, which has maximum error of about .015 and which is also simpler than x' to calculate.

Table 1

Comparison of the inverse cumulative normal function approximations x', x^*, and $\hat{x}$.

x	F(x)	x'	x^*	$\hat{x}$
0	.5	.0000	.0	-.0005
.1	.5398278	.1000	.0993	.1046
.3	.6179114	.3000	.2981	.3113
.5	.6914625	.5001	.4972	.5138
1.0	.8413447	1.0006*	.9975	1.0070
1.5	.9331928	1.5011	1.5024	1.4940
2.0	.9772499	1.9991	2.0093	1.9880
2.5	.9937903	2.4901	2.5093	2.4943
3.0	.9986501	2.9693	2.9873	3.0075
3.5	.9997674	3.4332	3.4266	3.5145
4.0	.9999683	3.8800	3.8128	3.9992

*The value .9939 shown in Page (1977) is incorrect.

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Abramowitz, M. and I. A. Stegun (eds) (1972) Handbook of Mathematical Functions, Washington, D.C.: U.S. Government Printing Office.

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Two approximations to the inverse distribution function of the normal distributions are given. Both approximations dominate, in terms of accuracy and case of implementation, a method recently proposed in the literature.

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